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Production Line Monitoring Using Mahalanobis-Taguchi System in Rubber-Based Product Industry

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ABSTRACT

The Mahalanobis–Taguchi System (MTS) is a multivariate diagnostic and projection method. Mahalanobis Distance (MD) is a calculation based on associations that can be defined and analysed for the base or the reference group between the variables and trends. Because of the reliability of prevision with limited, clustered data sets, the MTS is of interest. This is the form of data with commercial vehicle scores. This study focuses on control of the production line for the manufacture of rubber products. A collection of parameters is optimized by MTS, by defining the appropriate parameter impacting the quality of the product to boost product quality in production line management. The goal of this study is to reduce workplace dependency, to monitor progress in inventories due to quality problems and, in the meantime, to improve product quality.

1. Introduction

The classification methodology for diagnosis and prediction using multivariate data is Mahalanobis-Taguchi (MTS) which employs Mahalanobis distance as its assessment metric. Mahalanobis Distance (MD) is a metric focused on the differences between the variables and the trends, measurable and analyzable for the specific or reference unit [4]. Due to its reported precise provisioning using small, correlated data sets, MTS is of interest. Research is currently using the MD to consider measurable and distinct variables [3].

The measure given by MD is responsible for the correlation in a group and is crucial for MTS. The determination of the homogeneity of a population in a group cannot be done unless the interaction between the test components is considered [9]. With the most representative sample, the Mahalanobis Space (MS, the reference group), MD is created in the MTS. Using the orthogonal array (OA) and the signal-to-noise ratio (SNR) evaluation of the contribution of each attribute, the necessary attributes of the MTS are optimized once it is established. The outcome is an easy yet effective decision-making approach in multivariate issues [12].

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MD enables the identification of trends in multidimensional structures, one method of constructing condition monitoring systems. MTS provides a multidimensional framework for extracting information which has led to the use of the technique in the creation of defect identification, estimation, diagnosis, and forecasts. In condition control, several variables are typically assessed. By identifying the parameters that have strong links with the fault event, MTS significantly reduces the need to measure all parameters [11]. MD is defined in Eq. (1).

$$MD_j = D_j^2 = \frac{1}{k} Z_{ij}^T A^{-1} Z_{ij} \quad (1)$$

where k is the total number of variables; i the number of variables (ith1, 2..., k); j the number of samples (jth1, 2..., n); Zij the standardized vector of normalized characteristics of xij; xij the value of the i-th characteristic in the j-th observation; mi the mean of the i-th characteristic; si the standard deviation of the i-th characteristic; T the transpose of the vector; A-1 the inverse of the correlation matrix.

The range Mahalanobis is used to measure the resemblance between a recognized value set (normal group) and an unspecified value set (abnormal group). It was used effectively for a wide range of situations, primarily since it is highly sensitive to intervariable information adjustments [10]. Furthermore, given the fact that MD is measured by standard deviations from the mean of samples, a statistical measurement of how well an undetermined sample matches a recognized sample set is provided.

2. Methodology

In this study, the methodology of the research work to incorporate line monitoring into the MTS framework is presented. Quantitative approaches are used in this study. The experiment design is based on a production line in a rubber manufacturing company. Data is defined and collected to evaluate the parameters of the system by the method Mahalanobis Taguchi. The parameters are then tracked and evaluated using four fundamental stages of MTS through real-time data processing. A collection of tailored criteria for potential evaluation will be established throughout the production line with the aid of the program. The analysis is performed on-line quality inspection. After this process, the distance from Mahalanobis (MD) is determined to evaluate the relation between the parameters found and the quality of the product. This specifies the optimal parameters.

2.1 Research Development Stage

In the first phase of this stage, the overall parameters taken are identified. These parameters are then narrowed down based on the accuracy of the preset values. Some factors such as environmental factors, human error, machine errors and such can cause inconsistency in the data obtained. This may result in invalid MD values which may cause inaccuracy in the research conducted. In the second stage of phase 3, upon removing the external factors which may variate the validity of the obtained data, only a few parameters were identified as the critical parameters for this research. The identified parameters are then analyzed based on data collected on an hourly basis and analyzed using MTS analysis.

2.2 Research Validation Stage

The validation of the MTS line monitoring is proven by the product quality. Generally, the quality of the product is inspected by the Quality Assurance personnel based on a fixed standard. The passing rate and failure rate are determined by the acceptance and rejection which were found in a certain sampling quantity. The Acceptance Quality Level (AQL) indicates the final disposition of the product. Major visual, minor visual, and critical categories are non-cumulative defects. The final stage of the research confirms the effectiveness of implementing the MTS Line monitoring system in rubber-based manufacturing based on the quality inspection results.

2.3 Four Fundamental Stages of MTS Analysis

Mahalanobis Distance (MD) is a dimensionless distance metric focused on the similarity between the characteristics of the various patterns that can be defined and evaluated by the reference population. This reference population is referred to as Mahalanobis Space (MS) [8]. The distance measurement commonly referred to as the Mahalanobis Scale aids the discriminatory analysis approach to the assessment of the abnormality of datasets against the Mahalanobis Space.

MD is defined as in Eq. (2)

$$MD_j = D_j^2 = Z_{ij}^T C^{-1} Z_{ij} \quad \text{with } Z_{ij} = \frac{x_{ij} - m_i}{s_i} \tag{2}$$

where;
i is the number of variables (i = 1, 2, ... , k);
j is the number of samples (j = 1, 2, .., n);
Z_{ij} is the standardized vector of normalized characteristics of x_{ij};
x_{ij} is the value of the i^{th} character in the j^{th} observation;
m_i is the mean of the i^{th} characteristic;
s_i is the standard deviation of the i^{th} characteristic;
T is the transpose of the vector;
C^{-1} is the inverse of the correlation matrix

The MD formulation is scaled in the MTS technique where the current MD formulation has been specified by Eq. (2) and is divided by the term 'k' which represents the number of variables or features of the system.

Therefore, the equation for calculating the scaled MD in MTS methodology becomes

$$MD_j = D_j^2 = \frac{1}{k} Z_{ij}^T C^{-1} Z_{ij} \tag{3}$$

where; k = the number of characteristics/variables/items/features of samples

2.3.1 Stage 1: construction of a measurement scale

In order to construct a metric scale, a homogeneous data collection of common findings must be obtained in order to create a comparison category called a regular sample. It is used as the basis or reference point of the scale. Normal datasets collected need to be standardized in order to obtain a dimensionless unit vector followed by MD computing using Eq. (3).

The steps for the construction of MS are outlined below:
Calculate the mean characteristic in the normal data set as

$$\bar{x}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n} \quad (4)$$

Then, calculate the standard deviation for each characteristic

$$s_i = \sqrt{\frac{\sum_{j=1}^{n} (X_{ij} - \bar{x}_i)^2}{n-1}} \quad (5)$$

Next, normalize each characteristic and form the normalized data matrix (Zij) and its transpose Z_{ij}^{T}

$$Z_{ij} = \frac{(X_{ij} - \bar{x}_i)}{s_i} \quad (6)$$

Then, verify that the mean of the normalized data is zero

$$\bar{z}_i = \frac{\sum_{j=1}^{n} Z_{ij}}{n} = 0 \quad (7)$$

Verify that the standard deviation of the normalized data is one

$$s_z = \sqrt{\frac{\sum_{j=1}^{n} (Z_{ij} - \bar{z}_i)^2}{n-1}} = 1 \quad (8)$$

Form the correlation matrix (C) for normalized data. Calculate the element matrix (cij) as follows

$$c_{ij} = \frac{\sum_{m=1}^{n} (Z_m Z_{jm})}{n-1} \quad (9)$$

Compute the inverse matrix (C^{-1}),
Finally, calculate MDj as stated in Eq. (16).

2.3.2 Stage 2: assessment of the measurement scale

Observations outside of MS or abnormal data sets will be used to evaluate the measurement scale. Same statistical equation is replicated to determine the same target (MD value) using irregular sample results. However, irregular data is averaged on the basis of the mean and standard deviation of the average category and the reverse regression function of the usual data is often used during irregular MD.

2.3.3 Stage 3: feature selection to identify useful features

In the third step, the function extraction (optimization) is conducted on the device to isolate and pick usable features that are deemed to be efficient enough to execute the classification mission. In other terms, at this point, obsolete, unnecessary or disruptive features are eliminated. It is achieved using the simpler version of the orthogonal matrix structure called the Orthogonal Array (OA) as the

feature selection mechanism and using Signal-to-Noise Ratio (SNR) measurement metrics as the selection criterion [6]. The functions are allocated to the orthogonal two-stage testing list, 'used' as stage 1 and 'not used' as level 2. Redo MD calculation for each experiment performed with all associated variables on each irregular study. The MD value is then tabulated in the same experimental run format and, thus, the SNR for each run is then computed.

OA is a type of fractional Factorial Experimental Design (DOE) that differs from traditional full-factorial or fractional factorial DOE concepts in that it seeks to balance the combination or interaction of factors equally with a minimum number of experimental sequences [14]. The orthogonal array arrangement of the MTS is defined by the Latin symbology as La(bc) where L is Latin Square, a is the number of runs, b is the number of factor levels, and c is the number of factors. Table 1 illustrates an example of the OA structure of 7 factors with 8 numbers of runs under 2-factor levels.

Table 1
An OA structure of type L8(27) array

	Factor						
Run	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

To illustrate further, take OA in Table 2 as an example, take column 1 and column 2 as the pair columns in this OA structure, the number of repetitions of the level combinations in this pair is the same as depicted in Table 2 below:

Table 2
The number of repetitions of level combinations

		Number of repetitions			
Combinations		Col 1 & Col 2	Col 1 & Col 3	Col 1 & Col 7	Col 3 & Col 6
1	1	2	2	2	2
1	2	2	2	2	2
2	2	2	2	2	2
2	1	2	2	2	2

The same number of repetitions of level between any two columns of this OA structure will be obtained thus the L8(27) in Table 2 array can be said as orthogonal.

2.3.3.1 Signal to Noise Ratio (SNR)

SNR is described as measuring the accuracy of the measurement scale in order to predict irregular situations [13].

A higher SNR value means a lower prediction error and is expressed in decibels (dB) [11]. Therefore, SNR is used as an appraisal criterion to calculate the degree to which the importance of each component or function in the program relates to the potential of discrimination between the two. Therefore, features that do not include helpful details or offer more valuable information than the existing features (with respect to the distinction between usual and irregular samples) would be

excluded [1]. As a consequence, it would not only boost the precision of forecasts and speed up the analysis of results but will also enhance the awareness of the important features of the integrated method. SNR evaluation metric is often used to determine the efficiency of the integrated program and the degree of change in the output of the system without undertaking a major selection phase (function with an initial feature set) [4].

The larger the better SNR

$$\mathrm{SNR} = \eta_q = -10\log_{10};\frac{1}{t}\sum_{i=1}^{t}(\frac{1}{MD_i^2}>? \quad (10)$$

2.3.4 Stage 4: future deployment with useful features

The efficiency of the whole machine may also be calculated by using SNR values [7]. SNR refers to both variables (original combination, which is typically one of the OA combinations) and the appropriate range of variables (optimal combination, which might not be one of the OA combinations) are determined on the basis of MDs with pathological conditions. The increase in the value of the SNR shows the degree of improvement in the diagnosis process for the original system.

2.4 The Threshold Limiting

The Threshold Value (MDT) used in this case study is calculated based on an MD boundary region (BR) of the training set suggested by [2].

$$\mathrm{BR} = \mathrm{AxBMD_{Lower}} \leq \mathrm{MD(x)} \leq \mathrm{MD_{Upper}F} \quad (11)$$

with,

$$\mathrm{MD_{Lower}} = \frac{\mathrm{MD_{max}} - \mathrm{MD_{min}}}{2} \quad (12)$$

$$\mathrm{MD_{Lower}} = \frac{\mathrm{MD_{max}} - \mathrm{MD_{min}}}{2} \quad (13)$$

where MD_{max} and MD_{min} are the maximum and minimum MD(x) found in the training dataset. MD_T is obtained from $x \in BR$ such that it gives minimum Type I and Type II errors. Hence, using Eq. (11)-(13).

The theory of the approach is that, in the case of a two-classification question, regular observation may be considered positive and abnormal observation may be considered negative. As a rule, there will be four grouping findings:

i. TP (true positives) are positive cases that are correctly classified as positive cases.
ii. TN (true negatives) are negative cases which are correctly classified as negative cases.
iii. FP (false positives) are negative cases which are incorrectly classified; and negatively identified as positive cases.

Type I and Type II errors are described in Eq. (14) and (15), respectively. To determine the optimal threshold is to minimize the sum of Type I and Type II errors that minimize the shadow area in Figure 1.

$$\text{Type I error } \alpha = \frac{FN}{FN+TP} \tag{14}$$

$$\text{Type II error } \beta = \frac{FP}{TN+FP} \tag{15}$$

3. Results

3.1 Computation of Mahalanobis Distance (MD) Values with The Reference Sample

The 68 normal samples mentioned in the previous section were chosen as the reference sample and the MD value of each normal sample is computed. The steps to calculate the MD value for the first data is demonstrated as follows.

i. Calculate the mean characteristic in the normal data set

The mean (average) of each feature characteristic (each column of the dataset) is calculated. Taking the first feature characteristic (first column) of the dataset as an example, the mean of this feature characteristic is calculated.

$$\bar{x}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n} \tag{4}$$

Thus,

$$\bar{x}_1 = \frac{(26+26+26+\ldots\ldots\ldots\ldots\ldots+29+29+29)}{68} = 27.79$$

ii. The standard deviation for each characteristic is calculated according to Eq. (5)

$$s_i = \sqrt{\frac{\sum_{j=1}^{n} (X_{ij}-\bar{x}_i)^2}{n-1}} \tag{5}$$

$$s_1 = \sqrt{\frac{(26\text{-}27.79)^2+(26\text{-}27.79)^2+\ldots..(29\text{-}27.79)^2+(29\text{-}27.79)^2}{67}} = 0.79$$

Similar procedures of Step 1 and Step 2 of computing the mean and standard deviation of the rest of the columns (other characteristics) in the dataset were conducted. Table 3 depicts the computational results of means and standard deviations of those remaining characteristics.

Table 3
Means and standard deviations for all normal data

	A	B	C	D	E	F	G	H
1	26	52.0	22.0	35.8	36.0	118	118	118
2	26	52.0	22.0	35.8	36.0	118	118	118
3	26	52.0	22.0	35.8	36.0	118	118	118
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
66	29	52.0	27.0	38.6	37.5	119	119	119
67	29	52.0	26.0	38.6	37.5	120	120	120
68	29	52.0	26.0	38.6	37.5	120	120	120
Average, μ	27.79	52.00	24.65	32.17	36.60	121.15	120.82	120.44
Std Dev, σ	0.79	0.02	2.73	5.74	0.64	1.78	1.76	1.75

iii. The normalized data matrix (Zij) and then its transpose matrix (Z_ij^T) are formed; hence for the first characteristic (first column), the normalized value of the first data under this column is calculated as follows

$$Z_{11} = \frac{(X_{11} - \bar{x}_1)}{s_1} \tag{6}$$

$$= \frac{(26 - 27.79)}{0.79} = -2.27$$

The above calculation is applied to the very first data of the first column (first characteristic) in the dataset, a similar normalization procedure applies to the rest of the data (2nd data until the 68th data) on the remaining columns and their results are tabulated in Table 4.

iv. Then, the mean of the normalized data is verified to be close to zero. Take the first column, the verification is calculated as follows

$$\bar{z}_i = \frac{\sum_{j=1}^{n} Z_{ij}}{n} \tag{7}$$

$$= \frac{(-2.27 + -2.27 + -2.27 + \ldots + 1.52 + 1.52 + 1.52)}{68} = 0.00$$

v. The standard deviation of the normalized data is verified to be close to one. Take the first column against, the verification is calculated as follows

$$s_z = S\frac{\sum_{j=1}^{n} (Z_{ij} - \bar{z}_i)^2}{n - 1} \tag{8}$$

$$= S\frac{(-2.27-0.0)^2 + \ldots + (1.52-0.0)^2}{67}$$

Table 4 tabulated the results of step 4 and step 5 computations for all columns in the datasets. Note that only several numbers of samples are tabulated in the table due to page restriction.

Table 4
Normalized values for all normal data

	A	B	C	D	E	F	G	H
1	-2.26	0.21	-0.97	0.63	-0.94	-1.77	-1.60	-1.40
2	-2.26	0.21	-0.97	0.63	-0.94	-1.77	-1.60	-1.40
3	-2.26	0.21	-0.97	0.63	-0.94	-1.77	-1.60	-1.40
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
66	1.52	0.21	0.86	1.12	1.41	-1.21	-1.04	-0.82
67	1.52	0.21	0.50	1.12	1.41	-0.64	-0.47	-0.25
68	1.52	0.21	0.50	1.12	1.41	-0.64	-0.47	-0.25

vi. Correlation matrix (C) for the normalized data is formed. In this step, Excel built-in correlation function is deployed. Table 5 tabulated the result of the correlation matrix.

vii. The inverse matrix (C-1) is computed and Table 6 tabulates the inverse correlation matrix of this normal data set.

$$C^{-1} = \frac{1}{c_{ij}}$$

viii. Finally, MDj on each normal sample is calculated based on Eq. (3). Hence, the MD for the first sample is illustrated as follows

$$MD_1 = D_1^2 = \frac{1}{k} Z_{11}^T C^{-1} Z_{11}$$

$$= \frac{1}{8} \begin{bmatrix} -2.26 \\ 0.21 \\ -0.97 \\ \vdots \\ -1.77 \\ -1.60 \\ -1.40 \end{bmatrix} \begin{bmatrix} 3.16 & 0.30 & \cdots & -0.97 & -2.64 \\ 0.30 & 1.17 & \cdots & -0.29 & -0.28 \\ 2.13 & 0.73 & \cdots & -1.64 & -2.74 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 1.20 & 0.44 & \cdots & -4.92 & -5.69 \\ -0.97 & -0.29 & \cdots & 5.34 & 1.25 \\ -2.64 & -0.28 & \cdots & 1.25 & 7.26 \end{bmatrix} [-2.26 \ -0.21 \ -0.97 \ \ldots -1.77 \ -1.60 \ -1.40] = 1.44$$

Table 7 tabulated the MD values for the remaining 67 normal data (normal sample). These MD values are denoted as the reference space (unit space) of the MD measurement scale. Note that the grand average of the MDs of the normal samples is close to unity (one).

Table 5
Correlation matrix of the normal samples

	A	B	C	D	E	F	G	H
A	1.00	-0.06	0.16	-0.07	0.25	0.49	0.45	0.64
B	-0.06	1.00	-0.24	-0.22	-0.13	-0.02	0.02	-0.03
C	0.16	-0.24	1.00	0.79	0.89	-0.22	-0.22	-0.10
D	-0.07	-0.22	0.79	1.00	0.82	-0.50	-0.50	-0.41
E	0.25	-0.13	0.89	0.82	1.00	-0.34	-0.35	-0.22
F	0.49	-0.02	-0.22	-0.50	-0.34	1.00	0.89	0.88
G	0.45	0.02	-0.22	-0.50	-0.35	0.89	1.00	0.77
H	0.64	-0.03	-0.10	-0.41	-0.22	0.88	0.77	1.00

Table 6
Inverse correlation matrix

	A	B	C	D	E	F	G	H
A	3.16	0.30	2.13	0.58	-3.62	1.20	-0.97	-2.64
B	0.30	1.17	0.73	0.40	-0.91	0.44	-0.29	-0.28
C	2.13	0.73	7.49	-1.31	-6.63	1.62	-1.64	-2.74
D	0.58	0.40	-1.31	4.53	-2.34	0.09	0.45	0.46
E	-3.62	-0.91	-6.63	-2.34	10.32	-1.43	1.77	2.77
F	1.20	0.44	1.62	0.09	-1.43	9.75	-4.92	-5.69
G	-0.97	-0.29	-1.64	0.45	1.77	-4.92	5.34	1.25
H	-2.64	-0.28	-2.74	0.46	2.77	-5.69	1.25	7.26

Table 7
Normal MD values for the 68 Normal Data

	MD1	MD2	MD3	MD4	MD5	MD6	MD7	MD8	MD9	MD10	Average
0	1.44	1.44	1.44	0.44	0.44	0.44	0.44	0.44	0.44	0.48	0.74
1	0.48	0.48	0.48	0.71	0.71	0.69	0.69	0.69	0.69	0.74	0.64
2	0.74	0.87	0.87	0.87	0.74	0.74	0.74	0.74	0.70	0.70	0.77
3	0.70	0.70	0.70	0.70	0.79	1.52	1.52	0.78	2.32	3.01	1.28
4	1.05	0.67	0.67	1.44	1.10	0.99	0.99	0.84	0.86	0.54	0.91
5	2.54	1.25	1.25	1.25	1.79	2.73	0.53	0.37	0.55	1.32	1.36
6	2.90	0.42	0.58	2.73	0.58	1.06	0.87	0.87	Nil	Nil	1.25
									Grand Average		0.95

To assess the validity of the reference space, MD values for the 37 abnormal conditions were calculated. Similar computational procedures are implemented according to the previous sub-section starting from step (3) except the abnormal (failed samples) is used.

Each characteristic of the abnormal samples is normalized which eventually in total, a normalized data matrix (Z_{ij}) is formed, and then its transpose (Z_{ij}^{T}). For the first characteristic (first column), the normalized value of the first data under this abnormal dataset column is calculated as follows

$$Z_{11\ (abnormal)} = \frac{(X_{11(abnormal)} - \bar{x}_{1\ (normal)})}{s_{1\ (normal)}} \quad (6)$$

$$= \frac{(26 - 27.79)}{0.79} = -2.26$$

The important point to be aware of in this calculation is that the value of mean ($\bar{x}$ _i) and standard deviation (s_i) are based on the value of the normal samples corresponding to the abnormal data characteristic or column. Therefore, if the abnormal data from characteristic or column number 10

is used for instance, then the mean and standard deviation from column 10 of the normal dataset must be used. A similar normalization procedure applies to the rest of the abnormal training dataset (2nd data until the 37th data) on its remaining columns is tabulated in Table 9.

ix. Finally, the MD on each abnormal sample is computed using similar Eq. (16). The MD for the first abnormal sample is illustrated as follows

$$MD_{1\,(abnormal)} = D^2_{1(abnormal)} = \frac{1}{k} Z^T_{11(abnormal)} C^{-1}_{(normal)} Z_{11(abnormal)} \quad (16)$$

$$= \frac{1}{37} \begin{bmatrix} -2.26 \\ 0.21 \\ -0.97 \\ \vdots \\ -0.64 \\ -1.03 \\ -0.82 \end{bmatrix} \begin{bmatrix} 3.16 & 0.30 & \dots & -0.97 & -2.64 \\ 0.30 & 1.17 & & -0.29 & -0.28 \\ 2.13 & 0.73 & \dots & -1.64 & \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 1.20 & 0.44 & \dots & -4.92 & -5.69 \\ -0.97 & -0.29 & \dots & 5.34 & 1.25 \\ -2.64 & -0.28 & \dots & 1.25 & 7.26 \end{bmatrix} \begin{bmatrix} -2.74 \\ -0.28 \end{bmatrix} [-2.26 \;\; 0.21 \;\; -0.97 \dots -0.64 \;\; -1.03 \;\; -0.82] = 1.457$$

Thus, the MDs for the rest of the abnormal samples were calculated and the values were tabulated in Table 8.

Table 8
MD data for the 37 abnormal samples

Sample	MD_1	MD_2	MD_3	MD_4	MD_5	MD_6	MD_7	MD_8	MD_9	MD_{10}
0	1.46	1.46	1.46	1.47	1.47	1.47	1.47	1.34	0.81	0.87
1	1.52	1.19	0.84	3.01	3.01	3.01	3.01	3.01	3.01	3.01
2	3.01	1.44	1.44	0.84	0.84	0.84	0.86	0.81	2.54	1.25
3	1.79	0.97	0.85	0.82	0.82	0.87	0.87			

It could be observed that the MD values of the abnormal samples are larger than the normal samples. These MD values were then compared with MD values of the normal condition obtained in the previous section via a scatter plot to see the difference. It could be seen that there is a clear separation between normal and abnormal conditions based on their MD values. Thus, the discriminant ability of the reference space based on MD is assured.

Table 9
Normalized value for the abnormal samples

	A	B	C	D	E	F	G	H
1	-2.26	0.21	-0.97	0.67	-0.94	-0.64	-1.04	-0.82
2	-2.26	0.21	-0.97	0.67	-0.94	-0.64	-1.04	-0.82
3	-2.26	0.21	-0.97	0.67	-0.94	-0.64	-1.04	-0.82
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
35	0.26	0.21	0.86	1.10	1.41	-1.21	-1.60	-1.40
36	1.52	0.21	0.50	1.12	1.41	-0.64	-0.47	-0.25
37	1.52	0.21	0.50	1.12	1.41	-0.64	-0.47	-0.25

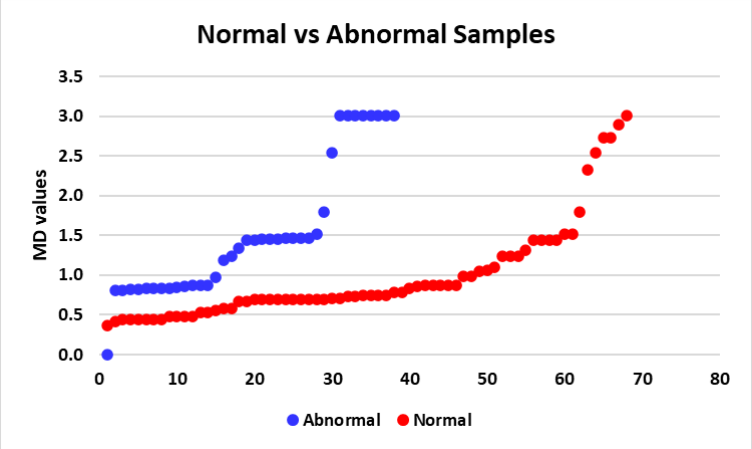


Fig. 1. Normal versus abnormal samples based on MD values

3.2 Threshold Limit Setting

For this case study, a threshold value of 0.8 (an MD value) denoted as MDT is selected based on Minimizing Type I and Type II errors computations. Minimizing Type I and Type II errors is a decision mechanism that integrates the risk factor into the judgment law where it tries to offset (minimize) the costs and losses incurred in making the decision.

The value of the MDT is crucial to any system of pattern recognition as it acts as a decision-making rule in the process of discrimination [15]. As in the sense of MTS, it is helpful to distinguish the value sample (i.e., check samples in this case) using an integrated method in the fourth stage of MTS. So, in this situation, the analysis, and the judgment rule should be specified in such a manner that the meaning of the test remains the same MD of a testing sample equals or less than MDT = 0.8, then the sample belongs to the normal group, otherwise, the sample belongs to the abnormal.

3.3 Identification of Significant Variables

The next step, after validation of the reference space, ensures that a useful range of variables is defined using the signal-to-noise ratio. The suggested algorithms would then be implemented utilizing a higher-than-better signal-to-noise ratio to reduce the usage of OA as in traditional MTS [5]. In the first case, though, the protocols would be preceded by the usage of the OA accompanied by the suggested algorithms.

3.4 Optimization by Orthogonal Array

In order to evaluate the suitable range of variables dependent on SNR, a correct orthogonal array (OA) must be chosen to minimize the number of variable variations. In this case, the orthogonal array L_{12} (2^{11}) is selected since there are 37 variables. The distribution of the elements in the chosen sequence is displayed in Table 10. Abnormal MDs are calculated for each run/combination. Because there are 37 abnormal samples, for each combination there are 37 MDs. As for a demonstration, MD computation for Run 2 is demonstrated as follows:

For Run 2, variables labelled ‘A’ until ‘E’ (5 variables) are denoted as use (‘1’) in the OA arrangement (see Table 10). Thus, the abnormal MD value corresponding to these 5 variables is calculated as follows:

First, the inverse correlation matrix of the normal samples based on 5 variables is computed and the correlation and its inverse matrix are presented in Table 11, 12 and 13.

$$MD_{1\ (Run\ 2)} = \frac{1}{k} {}^{1}Z^{T}_{11(5\ Variables)}\ C^{-1}_{(normal\ 5\ variables)}\ Z_{11\ (5\ variables)} \quad (3)$$

$$= \frac{1}{5}\begin{bmatrix}-2.26\\0.21\\\vdots\\0.63\\-0.94\end{bmatrix}\begin{bmatrix}1.43 & 0.19 & \cdots & 1.19 & -1.49\\0.19 & 1.14 & \cdots & 0.43 & -0.77\\\vdots & \vdots & \cdots & \vdots & \vdots\\1.19 & 1.19 & \cdots & 4.23 & -3.17\\-1.49 & -1.49 & & -3.17 & 7.59\end{bmatrix}[-2.26\ \ 0.21\ \ldots 0.63\ -0.94] = 1.75$$

Similar computational procedures are applied to the remaining abnormal samples corresponding to Run 2 of the OA. Eventually, the rest of the runs in following the 'use' variables indicated in every run of the OA were computed and the MDs corresponding to all runs in the OA were calculated as indicated in Table 10.

Next is the SNR computation on the MDs obtained corresponding to each run of the OA. Thus, using Eq. (10), the SNR for Run 2 is as follows

$$SNR = \eta_{Run\ 2} = -10\log_{10}\left(\frac{1}{5}\sum_{i=1}^{17}\left(\frac{1}{MD_i}\right)\right) \quad (10)$$

$$= -10\log_{10}\left(\frac{1}{5}\left(\frac{1}{1.46}+\frac{1}{1.46}+\ldots..+\frac{1}{0.87}+\frac{1}{0.87}\right)\right) = 1.01$$

Therefore, the SNR computations for the rest of the SNRs corresponding to the other OA runs are also presented in Table 11.

Table 10
Variable allocation in an L_{12} (2^{11}) orthogonal array

L12 Array	A	B	C	D	E	F	G	H	I	J	K
Run	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	2	1	2	2	1

Table 11
Correlation matrix of the normal samples based on 5 variables

	A	*B*	*C*	*D*	*E*
A	1.00	-0.06	0.16	-0.07	0.25
B	-0.06	1.00	-0.24	-0.22	-0.13
C	0.16	-0.24	1.00	0.79	0.89
D	-0.07	-0.22	0.79	1.00	0.82
E	0.25	-0.13	0.89	0.82	1.00

Table 12
Inverse correlation matrix of the normal samples based on 5 variables

	A	B	C	D	E
A	1.43	0.19	0.21	1.20	-1.50
B	0.19	1.14	0.59	0.43	-0.77
C	0.21	0.59	5.30	-0.59	-4.20
D	1.20	0.43	-0.59	4.23	-3.18
E	-1.50	-0.77	-4.20	-3.18	7.60

Table 13
Inverse correlation matrix of the normal samples based on 5 variables

	MD_1	MD_2	MD_3	MD_4	MD_5	...	MD_{33}	MD_{34}	MD_{35}	MD_{36}	MD_{37}	SNR
R1	1.46	1.46	1.46	1.47	1.47	...	0.85	0.82	0.82	0.87	0.87	1.01
R2	1.81	1.81	1.81	1.81	1.81	...	0.80	0.55	0.55	1.29	1.29	0.28
R3	1.32	1.32	1.32	1.50	1.50	...	0.59	1.25	1.25	1.03	1.03	-0.71
R4	1.88	1.88	1.88	2.19	2.19	...	1.96	0.84	0.84	1.66	1.66	0.62
R5	1.83	1.83	1.83	1.86	1.86	...	1.02	1.27	1.27	1.47	1.47	0.77
R6	1.92	1.92	1.92	1.77	1.77	...	1.88	1.45	1.45	1.64	1.64	-0.34
R7	0.64	0.64	0.64	1.93	1.93	...	0.35	0.90	0.90	0.72	0.72	-2.21
R8	0.30	0.30	0.30	0.67	0.67	...	0.50	0.80	0.80	0.52	0.52	-3.14
R9	0.86	0.86	0.86	1.47	1.47	...	0.65	1.00	1.00	0.16	0.16	-2.05
R10	2.01	2.01	2.01	2.21	2.21	...	0.53	0.99	0.99	0.54	0.54	0.71
R11	0.76	0.76	0.76	1.30	1.30	...	0.65	0.96	0.96	1.21	1.21	0.00
R12	1.62	1.62	1.62	2.02	2.02	...	0.60	0.64	0.64	0.58	0.58	0.20

The average result (in terms of SNRs) for all variables is shown in Table 14. These answers are determined for two ranges (l's and 2's) of the variables allocated to separate OA columns. For instance, the average response of X1 at level 1 is the ratio of the sum of the signal-to-noise ratios corresponding to l of X1 (column 1) to the total number of l's. The gain in SNR for a variable is the difference between the average responses of that variable at level 1 and level 2. Figure 2 illustrates the average responses plot for all features. A positive gain signifies the importance of the variable.

Table 14
Average responses of the Larger-the-Better S/N Ratios

	A	B	C	D	E	F	G	H
Average SNR 1	0.27	-1.14	0.01	-0.03	-0.09	-0.06	-0.04	-0.50
Average SNR 2	-1.08	0.33	-0.82	-0.78	-0.72	-0.75	-0.77	-0.31
Gain (+/-)	1.35	-1.47	0.83	0.76	0.63	0.68	0.72	-0.18

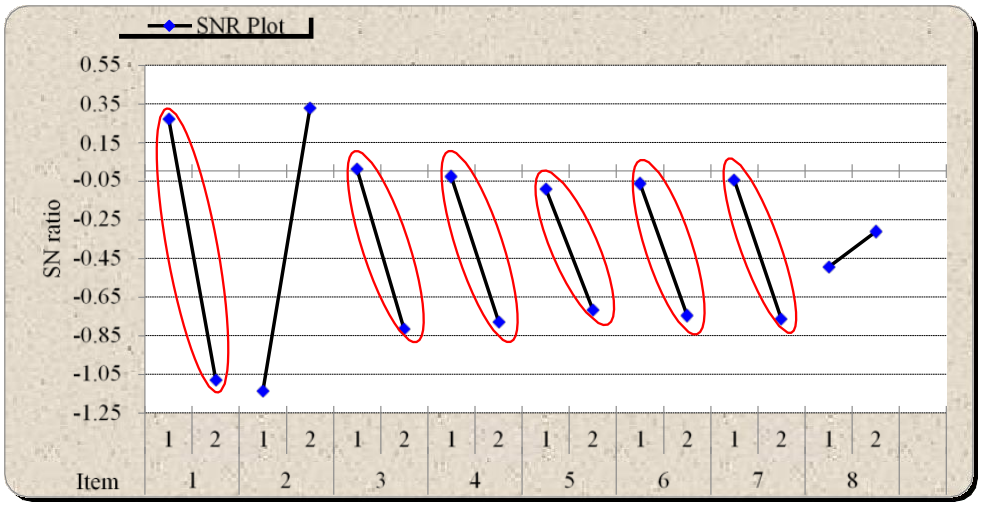


Fig. 2. Average responses plot

From Table 14 and Figure 1, it was found that features labelled 'A', 'C', 'D', 'E', 'F' and 'G' (marked in red oval) showed positive gains and hence they are considered as useful variables by the OA. This constitutes a reduction of 25% of the original features.

Thus, it was observed that the deployment of OA in optimizing the MTS managed to reduce (optimize) the number of parameters needed to obtain a PASS sample from 8 parameters to only 6 parameters without jeopardizing the performance based on the SNR gain value. In fact, with only 6 parameters, an improvement in the reduced diagnosis system SNR was observed with a positive SNR gain indicating 'less variability' of the optimized system for better predictability in classifying future prediction tasks using MD measurements.

3.5 Real-Time Line Monitoring

From the previous stages, the significant parameters which affect the quality of the product have been identified based on the data collection activity conducted. In this stage, a real-time implementation or monitoring is carried out to prevent the occurrence of future abnormalities by benchmarking the MDT value calculated in Stage 2. The MDT =0.8 was determined as the discriminant to identify if the collected data would produce a normal or abnormal product. Thus, in this stage, a series of data is collected every 30-minute interval from the start of the production line running until the MD value is found to be close to the calculated MDT value which indicates the prominent failure of the product indicating the technician to check and adjust the significant parameters identified. Table 15 shows the data collected in real-time.

Table 15
Real timeline monitoring

	A	B	C	D	E	F	G	H
8.00 am	28.5	52.0	22.0	24.0	35.8	124.0	123.0	123.0
8.30 am	27.5	52.0	22.0	24.0	36.0	123.0	123.0	122.0
9.00 am	29.0	52.0	22.0	24.0	36.0	123.0	123.0	123.0
…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…
1.30 pm	27.5	52.0	22.0	24.0	36.0	122.0	122.0	122.0
2.00 pm	28.5	52.0	22.0	24.0	36.0	124.0	123.0	123.0
2.30 pm	28.5	52.0	22.0	24.0	36.0	123.0	123.0	123.0

Following the same calculation method used in stage 1, the MD values are calculated at every 30 minutes interval as shown in Table 16. The trend of the parameters is then plotted as in Figure 3.

Table 16
Calculated MD values during real-time monitoring

Sample no.	Time	MD value
1	8.00 am	0.52
2	8.30 am	0.66
3	9.00 am	0.63
4	9.30 am	0.70
5	10.00 am	0.70
6	10.30 am	0.70
7	11.00 am	0.74
8	11.30 am	0.74
9	12.00 pm	0.75
10	12.30 pm	0.76
11	1.00 pm	0.77
12	1.30 pm	0.80
13	2.00 pm	0.44
14	2.30 pm	0.54

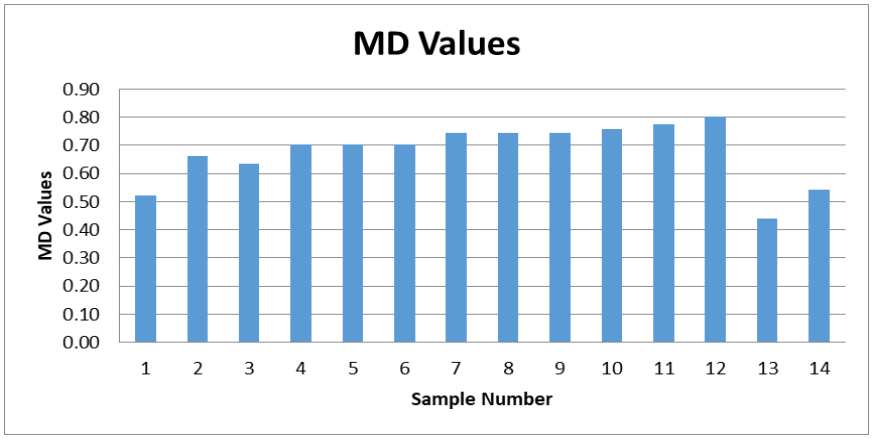


Fig. 3. Real-time MD Value in comparison with MDT

At the beginning stage of the production line when the technician sets the parameters, it shows a value lower than the threshold MD value which is 0.8. Meaning to say from 8.00 am up to 10.30 am the line was in a stable condition which was evaluated by the quality inspection to verify as well. At 11.00 am onwards the production line showed some deterioration in quality as in the initial stage of failures were identified, which was then reflected by the MD value. The pre-quality inspection which is usually done by the line technicians verified the MD value obtained.

4. Conclusions

Reliable pattern analysis was presented based on the four fundamental stages of MTS monitoring. The real-time monitoring and implementation to replace the conventional after effect troubleshooting activity was successfully demonstrated and validated. The validation test on the testing dataset confirms the effective line monitoring system to be implemented in the production line. By this monitoring system, the production staff can alert the technician to troubleshoot the system before the system fails and to prevent future occurrences. Despite the previous practice, the production staff was unsure of which parameter to troubleshoot when there was a failure found.

However, through the SNR gain, optimum parameters contributing to the life efficiency and quality of the product after being produced are optimized. Thus, through this MTS analysis system, the production technician would be able to troubleshoot before the failure with the identified significant parameters.

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